Centerline Velocity Decay of Compressible Free Jets

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Theme

An empirically determined formulation for the eddy viscosity is introduced into the turbulent axially symmetric compressible free jet theory of Kleinstein.[1] The easily evaluated algebraic equation that results shows excellent agreement with an extensive compilation of experimental data for the centerline velocity decay in the main region. It is found that the eddy viscosity can be treated as constant, with functional dependence on only the exit Mach number and the jet-to-freestream density ratio. The theory presented is applicable to the main region of most classes of free jets, including heterogeneous, nonisothermal, and subsonic and properly-expanded supersonic flows.

Contents

Numerous theories that use the phenomenological approach to describe the turbulent mixing region of axially symmetric compressible free jets have been published. In each instance a limited amount of experimental data was used to determine empirical constants in the models for the eddy diffusivities. The study presented in this report selected one of the more promising of these theories, and compared it to an extensive compilation of experimental results for the decay of centerline velocity. The result is a generalized linear expression that correlates well with the full range of experimental investigations.

The theoretical approach chosen for this study was first applied to laminar jet flows by Kleinstein.[2] The method avoids the assumption of flow similarity by the approximate technique of linearizing the diffusion terms of the boundary-layer equations in the plane of the von Mises transformation. More recently, Kleinstein[1] presented an analysis for turbulent axially symmetric compressible flow that was based on a compressible formulation of the eddy viscosity suggested by Ferri, Libby, and Zakkay[3]

$$\rho\varepsilon = (\kappa/4) b \rho_c u_c$$

where ρ is density, ε the eddy viscosity, κ a proportionality constant, b the jet half-width, and u the mean velocity, with the subscript c referring to values on the jet centerline. When this form for the eddy viscosity is introduced into the linearized momentum equation, it is found that the eddy viscosity for free jets is a constant, having the form

$$\rho\varepsilon = (\kappa/4)(\bar{\rho}_e)^{0.5} \rho_j u_j r_j$$

where r is the radial coordinate, and $\bar{\rho}_e = \rho_e/\rho_j$, with j and e referring to quantities at the jet exit and the external conditions, respectively. The resulting expression for centerline velocity decay can then be shown to be

$$\bar{u}_c(\bar{x}) = 1 - \exp\left(\frac{-1}{\kappa\bar{x}(\bar{\rho}_e)^{0.5} - X_c}\right) \quad (1)$$

Here, x is the axial coordinate, and $\bar{x} = x/r_j$. X_c is the nondimensional correlation parameter core length, which Kleinstein found to have a universal value of 0.70. Thus, the length of the potential core is

$$\bar{x}_{\text{core}} = 0.70/\kappa(\bar{\rho}_e)^{0.5}$$

Using the heated-air into ambient-air jet mixing data of Corrsin and Uberoi,[4] Kleinstein concluded that $\kappa = 0.074$, a universal constant.

For the present study, the proportionality constant κ was found by first rearranging Eq. (1) to give κ as an explicit function of the measurable quantities $\bar{u}_c$, $\bar{x}$, and $\bar{\rho}_e$, with X_c taken to be 0.70. The data of the experiments summarized in Table 1 was next

Table 1 Free jet centerline velocity experiments

Experimenter[a]	Jet composition	M_j	$\bar{\rho}_e$	Re_j^b
Alexander, Baron, & Comings	Ambient air	0.160	1.00	7.77×10^4
(1949)	Ambient air	0.360	1.00	1.82×10^5
	Ambient air	0.710	1.00	3.62×10^5
Corrsin & Uberoi (1950)	Heated air	0.067	1.05	3.55×10^4
	Heated air	0.067	1.58	2.20×10^4
	Heated air	0.067	2.02	1.65×10^4
Keagy & Weller (1949)	Cooled He	0.120	7.25	2.65×10^3
	Cooled N_2	0.350	1.01	2.11×10^4
	Cooled CO_2	0.460	0.64	4.02×10^4
Snedecker & Donaldson (1964)	Cooled air	0.213	0.99	6.15×10^4
	Cooled air	0.516	0.95	1.50×10^5
	Cooled air	0.962	0.84	3.40×10^5
Taylor, Grimmett, & Comings	Ambient air	0.159	1.00	6.78×10^4
(1951)	Ambient air	0.345	1.00	1.76×10^5
Tomich (1967)	Heated air	0.599	2.81	2.46×10^4
	Heated air	0.625	2.70	3.93×10^4
	Heated air	0.630	2.72	3.93×10^4
	Heated air	0.631	2.72	3.99×10^4
	Heated air	0.753	2.21	4.01×10^4
	Heated air	0.755	2.68	3.26×10^4
	Heated air	0.799	2.91	2.94×10^4
	Heated air	0.837	2.62	3.71×10^4
Warren (1957)	Heated air	0.680	1.31	7.04×10^5
	Cooled air	0.690	0.91	1.12×10^6
	Heated air	0.690	1.13	8.30×10^5
	Cooled air	0.967	0.84	1.76×10^6
	Heated air	0.967	1.04	1.32×10^6
	Heated air	0.970	1.20	1.11×10^6
Wygnanski & Fiedler (1969)	Ambient air	0.150	1.00	8.73×10^4
Broer & Rietdijk (1960)	Cooled air	1.74	0.62	1.69×10^6
Eggers (1966)	Cooled air	2.22	0.50	3.14×10^6
Johannesen (1959)	Cooled air	1.40	0.72	9.01×10^5
Pitkin and Glassman (1958)	Cooled air	2.60	0.42	2.22×10^7
Warren (1957)	Cooled air	2.60	0.42	2.22×10^7

[a] Complete citation is given in the full paper.
[b] Jet Reynolds number based on nozzle diameter.

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Index category: Jets, Wakes, and Viscid-Inviscid Flow Interactions.

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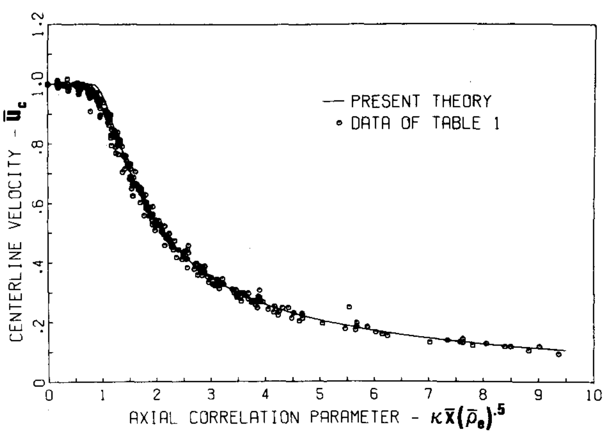

Fig. 1 Subsonic free jet centerline velocity data compared to theory.

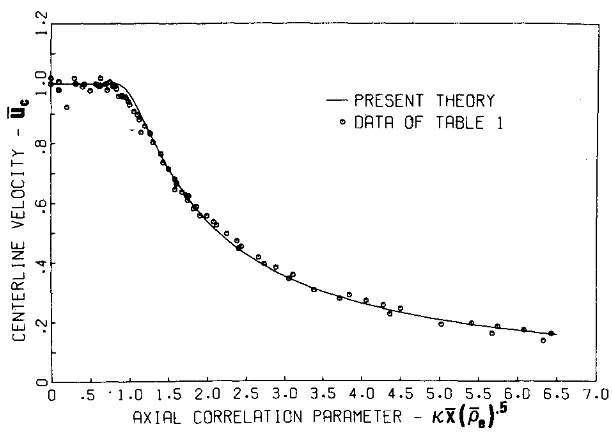

Fig. 2 Supersonic free jet centerline data compared to theory.

used to evaluate κ for a wide range of conditions. By then plotting κ as a function of $\bar{\rho}_e$ and the jet Mach number, M_j, it was found that

$$\kappa = 0.08(1-0.16M_j)(\bar{\rho}_e)^{-0.22} \tag{2}$$

The linear dependence of the eddy viscosity on the Mach number is identical to that found by Warren,[4] whereas the density dependence appears to be unique to the theory of Kleinstein.† The data from the experiments listed in Table 1 is compared against Eqs. (1) and (2) in Fig. 1. The excellent agreement with theory is evident.

From the supersonic jet experiments shown in Table 1 (restricted to properly expanded nozzles) it was found that the eddy viscosity in the supersonic region is different from that of the downstream subsonic region. Such a two-region model was proposed by Anderson and Johns.[5] For the present study it was found that the theory of Kleinstein correlated well with data for the following value of the proportionality constant:

$$\kappa = 0.063(M_j^2-1)^{-0.15} \tag{3}$$

This formulation is applicable from the jet exit to the sonic point, which is specifically the region

$$M_j[\bar{u}_c/(\bar{T}_c)^{1/2}] \geqq 1$$

with $\bar{T}_c = T_c/T_j$, where T is the static temperature. An expression for the centerline decay of total enthalpy, H, is given in Ref. 1 as

$$\frac{H_c-H_e}{H_j-H_e} = 1-\exp\left[\frac{-1}{0.102\bar{x}(\bar{\rho}_e)^{0.5}-0.70}\right] \tag{4}$$

The simultaneous solution of Eqs. (1) and (4) can be performed to locate the sonic point. Using the requirement of continuity of velocity, a virtual exit for the subsonic jet that begins at the sonic point can then be located by the equation

$$\Delta\bar{x}_{M=1} = \bar{x}_{M=1}\left(1-\frac{\kappa_j}{\kappa_{M=1}}\right)$$

from which an adjusted axial position can be defined as

$$\bar{x}' = \bar{x} - \Delta\bar{x}_{M=1}$$

Thus, by replacing $\bar{x}$ with $\bar{x}'$, and setting $M_j = 1$, Eq. (1) can be solved for the subsonic region. It was also found to be sufficient to assume that the supersonic jet density-ratio is applicable at the virtual exit of the subsonic region. The results achieved with this approach are given in Fig. 2 for the data listed in Table 1. Considering the unlikely existence of perfectly expanded nozzles, agreement is extremely good.

† Because the derivation of Eq. (1) assumes that κ is a constant, it is not possible here to look for an eddy model that depends on the local Mach number, such as was proposed by Donaldson and Gray.[6]

References

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[2] Kleinstein, G., "An Approximate Solution for the Axisymmetric Jet of a Laminar Compressible Fluid," *Quarterly of Applied Mathematics*, Vol. 20, No. 1, April 1962, pp. 49–59.

[3] Ferri, A., Libby, P. A., and Zakkay, V., "Theoretical and Experimental Investigation of Supersonic Combustion," ARL 62-467, Sept. 1962, Aeronautical Research Labs., Wright-Patterson Air Force Base, Ohio.

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